Lec 7 — Alg design techniques continued

More divide and conquer

Generally...
- Base case ...
- Inductive case
  1. Divide into $f(n)$ parts of $g(n)$ size
  2. Recurse
  3. Combine results

Skeleton

```
DC A =
  if |A| = 0  =>  ----
  if |A| = 1  =>  ----
  else let
    (L,R) = DC ( A[0, |A|/2] || B[ |A|/2, |A|] )
  in
    combine (L,R)
  end
```

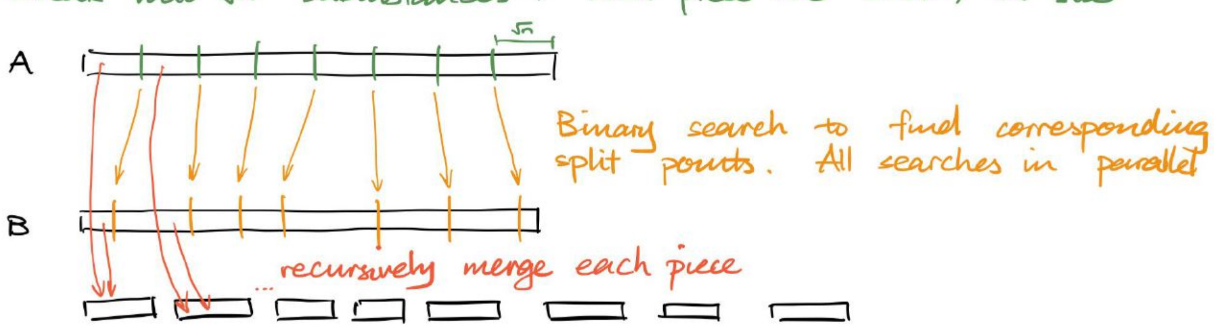
.. but that's long we can actually do

reduce combine empty $\langle$ base (x) $|$ $x \in A$ $\rangle$

Merge with $O(n)$ work $O(\log n)$ span

Let $n = |A| + |B|$, WLOG $|A| > |B|$

Break into $\sqrt{n}$ subinstances. Each piece also $O(\sqrt{n})$ in size



Asumming ...

$W_{split} = O(\sqrt{n} \lg n)$ $\quad$ $S_{split} = O(\lg n)$

$W_{combine} = O(\sqrt{n})$ $\quad$ $S_{combine} = O(\lg n)$

(for some tree sequence impl)

Then overall ...

this is assuming B corresponds to A's split well

$$W(n) = \sqrt{n}\, W(\sqrt{n}) + W_{split}(n) + W_{combine}(n)$$
$$= \sqrt{n}\, W(\sqrt{n}) + O(\sqrt{n} \lg n)$$

parent $\quad \sqrt{n} \log n$

children $\quad \sqrt{n} \cdot (\sqrt{\sqrt{n}} \log \sqrt{n}) = n^{\frac{3}{4}} \cdot \frac{1}{2} \cdot \lg n$ $\quad$ Leave dominated

... $W(n) \in O(n)$

$$S(n) = S(\sqrt{n}) + S_{split}(n) + S_{combine}(n)$$
$$= S(\sqrt{n}) + O(\lg n)$$

parent $\quad \lg n$

child $\quad \lg \sqrt{n} = \frac{1}{2} \lg n$ $\quad$ Root dominated

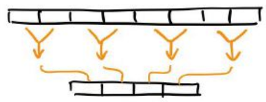
So $S(n) \in O(\lg n)$

Contraction

Break into one piece ... but recursively solve the one piece

- Base case ...
- Inductive case
  1. Contract into one piece of size $g(n)$
  2. Recurse on subinstance
  3. Expand result to solve original problem

Ex. reduce f I S = cas |S| of
  0 ⇒ I
  1 ⇒ f(I, S[0])
  _ ⇒ let
    $B = \langle f(S[2i], S[2i+1]) \mid 0 \le i < \frac{|S|}{2} \rangle$
  in
    reduce f I B
  end



↓ Shorter subproblems...

$$W(n) = W(\tfrac{n}{2}) + O(n) \quad \in O(n)$$

$$S(n) = S(\tfrac{n}{2}) + O(1) \quad \in O(\lg n)$$

Ex. scan [omitted]

scan op+ 0 ⟨2, 1, 1, 4, 3, 2, 1, 3⟩

contract: 3, 5, 5, 4

recursive scan (⟨0, 3, 8, 13⟩, 17)

expand (⟨0, 2, 3, 4, 8, 11, 13, 14⟩, 17)

$$W(n) = W(\tfrac{n}{2}) + O(n) \quad \in O(n)$$

$$S(n) = S(\tfrac{n}{2}) + O(1) \quad \in O(\lg n)$$