

Lec 0

Inductive Definitions

Writing things down inductively
 \hookrightarrow CD specs, rules, ...

Judgements - like asserting, types,

- "n is even"
 - e has type τ
 - live(l, x) - x is alive at line l
- Notation: $(l, x) \in L \Leftrightarrow \text{live}(l, x)$

Arbitrary predicate $p(n) \ n \in \mathbb{N} \rightarrow$ induction

Inference rule notations

if none, this is an "axiom"

$$\frac{}{\text{nat}(0)} \text{No}$$

$$\frac{\text{nat}(n)}{\text{nat}(n+1)} \text{N}_1$$

\leftarrow LHS of implication "premises"
 \leftarrow RHS of implication "conclusion"

$$\frac{}{\text{leq}(0,0)} \text{leq}_0 \quad \frac{\text{leq}(n,m)}{\text{leq}(n+1,m+1)} \text{leq}_1 \quad \frac{\text{leq}(n,m)}{\text{leq}(n,m+1)} \text{leq}_2$$

$$\frac{}{\text{even}(0)} \text{Even}_0 \quad \frac{\text{even}(n)}{\text{odd}(n+1)} \text{Odd}_1 \quad \frac{\text{odd}(n)}{\text{even}(n+1)} \text{Even}_1$$

Rule chaining

Ex. proving $\text{nat}(3) \Leftrightarrow 3 \in \mathbb{N}$

$$\frac{\frac{\frac{}{\text{nat}(0)} \text{No}}{\text{nat}(0+1)} \text{N}_1}{\text{nat}((0+1)+1)} \text{N}_1}{\text{nat}((0+1)+1)+1} \text{N}_1$$

Ex. proving 2 even

$$\frac{\frac{\frac{}{\text{even}(0)} \text{Even}_0}{\text{odd}(1)} \text{Odd}_1}{\text{even}(2)} \text{Even}_1$$

Ex. try prove 3 even ...

$$\frac{\frac{\text{odd}(0)}{\text{even}(1)} \leftarrow \text{no rule to pattern match on here}}{\text{odd}(2)} \leftarrow \text{no rule to pattern match on here}}{\text{even}(3)}$$

Notes

- usually multiple ways to get to a conclusion in that case pick one
- \rightarrow try not to let this happen!
- \rightarrow or implement backtracking
- Saturation: start with axioms, try derive everything

Try: $\frac{\text{odd}(n)}{\text{odd}((n+1)+1)} \text{Odd}_*$

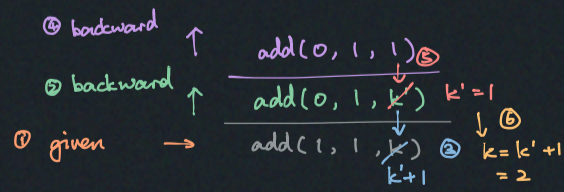
Mode of judgement

Given $n, \text{leq}(n, m)$, find what m works

Consider: $\text{add}(a, b, c) \equiv a+b=c$

$$\frac{}{\text{add}(0, n, n)} A_1 \quad \frac{\text{add}(m, n, k)}{\text{add}(m+1, n, k+1)} A_2$$

\leftarrow These define a set $(m, n, k) \in \mathbb{Z}^3$



Computing the set of a judgement

Ex. set of $(i, k) \in \mathbb{Z}^2, i, k \leq 2, \text{leq}(i, k)$

- start \emptyset
- apply axiom $\{(0, 0)\}$
- apply inference $\{(1, 1), (0, 1), (0, 0)\}$
- continue $\{(2, 2), (1, 2), (0, 2), (1, 1), (0, 1), (0, 0)\}$