Lec 0 Inductive Definitions

Writing things down inductively
└ cO specs, rules, ...

Judgements — like asserting, types,

- "n is even"
- e has type τ
- live (ℓ, x) — x is alive at line ℓ

Notation: $(\ell, x) \in L \Leftrightarrow$ live(ℓ, x)

Arbitrary predicate p(n) $n \in \mathbb{N}$ → induction

Inference rule notations

if none, this is an "axiom"

$$\frac{}{\text{nat}(0)}\ \mathbb{N}_0 \qquad \frac{\text{nat}(n)}{\text{nat}(n+1)}\ \mathbb{N}_1$$

← LHS of implication "premises"
← RHS of implication "conclusion"

$$\frac{}{\text{leq}(0,0)}\ \text{leq}_0 \qquad \frac{\text{leq}(n,m)}{\text{leq}(n+1,m+1)}\ \text{leq}_1 \qquad \frac{\text{leq}(n,m)}{\text{leq}(n,m+1)}\ \text{leq}_2$$

$$\frac{}{\text{even}(0)}\ \text{Even}_0 \qquad \frac{\text{even}(n)}{\text{odd}(n+1)}\ \text{Odd}_1 \qquad \frac{\text{odd}(n)}{\text{even}(n+1)}\ \text{Even}_1$$

Rule chaining

Ex. proving nat(3) ⇔ $3 \in \mathbb{N}$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\text{nat}(0)}\ \mathbb{N}_0}{\text{nat}(0+1)}\ \mathbb{N}_1}{\text{nat}((0+1)+1)}\ \mathbb{N}_1}{\text{nat}(((0+1)+1)+1)}\ \mathbb{N}_1$$

Ex. proving 2 even

$$\dfrac{\dfrac{\dfrac{}{\text{even}(0)}\ \text{Even}_0}{\text{odd}(1)}\ \text{Odd}_1}{\text{even}(2)}\ \text{Even}_1$$

Ex. try prove 3 even ...

$$\dfrac{\dfrac{\dfrac{\text{odd}(0)}{\text{even}(1)}}{\text{odd}(2)}}{\text{even}(3)}$$

← no rule to pattern match on here

Notes
- usually multiple ways to get to a conclusion in that case pick one
  → try not to let this happen!
  → or implement backtracking
- Saturation: start with axioms, try derive everything

Try: $\dfrac{\text{odd}(n)}{\text{odd}((n+1)+1)}\ \text{Odd}_*$

Mode of judgement

Given n, leq(n, m), find what m works

Consider: add(a, b, c) ≡ a+b=c

$$\frac{}{\text{add}(0,n,n)}\ A_1 \qquad \frac{\text{add}(m,n,k)}{\text{add}(m+1,n,k+1)}\ A_2$$

← These define a set $(m,n,k) \in \mathbb{Z}^3$

③ backward ↑ add(0, 1, 1) ⑤
② backward ↑ add(0, 1, k') k' = 1 ↓ ④ ⑥
① given → add(1, 1, k) ② k = k'+1 = 2

Computing the set of a judgement

Ex. set of $(i,k) \in \mathbb{Z}^2$, $i,k \le 2$, leq(i,k)

① start ∅
② apply axiom {(0,0)}
③ apply inference {(1,1), (0,1), (0,0)}
④ continue {(2,2), (1,2), (0,2), (1,1), (0,1), (0,0)}