21-241

Matrices and Linear Transformations

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At Carnegie Mellon University

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Lec 1

- Go read syllabus
- Go get textbook
- Go do canvas HW#1

The Fundamental Problem

→ Solving systems of linear equations

$$\begin{matrix} a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ \vdots & & \vdots & & \ddots & & \vdots & & \vdots \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & = & b_2 \end{matrix}$$

They get hard if the system gets big.

Equivalence

Def: two systems equivalent if they have same solution.

Allowed operations

1. Change order of equations
2. Multiply an equation by non-zero constant
3. Add multiple of one equation to another

Ex.

Easier to solve

$$\begin{cases} x+y=3 \\ x-\frac{1}{2}y=3 \end{cases} \xrightarrow{(1)\mapsto(1)+(2)(2)} \begin{cases} 3x+0y=9 \\ x-\frac{1}{2}y=3 \end{cases}$$

$$\begin{cases} 3x+0y=9 \\ x-\frac{1}{2}y=3 \end{cases} \xrightarrow{(1)\mapsto(\frac{1}{3})(1)} \begin{cases} x \quad\quad = 3 \\ x-\frac{1}{2}y=3 \end{cases}$$

$\Rightarrow y=0.$ So $(x,y) = (3,0)$

Num of Solutions

- None
- One unique solution
- Infinite num of solutions

Why does operation 3 not change solutions? (not formal)

WTS: sol to old sys must be sol to new sys.
Suppose $(x, y) = (a, b)$ is a sol

Then
$x + y = 3$
$x - \frac{1}{2}y = 3$
$(x+y) + 2(x - \frac{1}{2}y) = 3 + 2(3)$ ← new equation

But
$a + b = 3$
$a - \frac{1}{2}b = 3$
So $(a+b) + 2(a - \frac{1}{2}b) = 9$.

WTS: new solution doesn't have extra solution.
Idea: any type 3 solution can be reversed with another type 3 operation.

$$\begin{cases} x + y = 3 \\ x - \frac{1}{2}y = 3 \end{cases} \xrightarrow{(1) \mapsto (1) + (2)(2)} \begin{cases} 3x + 0y = 9 \\ x - \frac{1}{2}y = 3 \end{cases}$$

$$\begin{cases} 3x + 0y = 9 \\ x - \frac{1}{2}y = 3 \end{cases} \xrightarrow{(1) \mapsto (1) + (-2)(2)} \begin{cases} x + y = 3 \\ x - \frac{1}{2}y = 3 \end{cases}$$

How to make syss easier to solve?

Trying to isolate one variable in each row.

$$\begin{aligned} x - y - z &= 1 \\ 2x - 3y - z &= 3 \\ -x + y - z &= -3 \end{aligned} \xrightarrow{(2) \mapsto (2) + (-2)(1)} \begin{aligned} x - y - z &= 1 \\ -y + z &= 1 \\ -x + y - z &= -3 \end{aligned}$$

$\downarrow$ $(3) \mapsto (3) + (1)(1)$

$$\begin{aligned} x - y - z &= 1 \\ -y + z &= 1 \\ z &= 1 \end{aligned} \xleftarrow{(3) \mapsto (-\frac{1}{2})(3)} \begin{aligned} x - y - z &= 1 \\ -y + z &= 1 \\ -2z &= -2 \end{aligned}$$

$\downarrow$ $(2) \mapsto (2) + (-1)(3)$, $(1) \mapsto (1) + (1)(3)$

$$\begin{aligned} x - y \quad &= 2 \\ -y \quad &= 0 \\ z &= 1 \end{aligned} \quad \cdots\cdots\to \quad \begin{aligned} x \quad &= 2 \\ y \quad &= 0 \\ z &= 1 \end{aligned}$$