Lec 2

username [andrew id]
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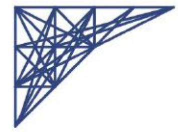

- Do WebWork orientation + WW#2
  └ 5 attempts
- Do HW#2

Seen systems with:

- One solution
- No solution
- Infinitely many solutions
  └ $(x, y, z) = (t+2, t, 1)$ for some $t \in \mathbb{R}$ — Called "parametric solution"

  ... or write $(x, x-2, 1)$
  $(y+2, y, 1)$
  $\vdots$

} Only these 3 possibilities

Making choices

* Leading variable — first variable in equation

$$x - y = 2$$
$$z = 1$$

O — leading variable
O — "free" variable ← Can choose value.

Matrix representation

$$\begin{aligned} 2x - 5y + z &= 6 \\ x + 2y - z &= 3 \\ -x + y + 2z &= 5 \end{aligned} \quad \longleftrightarrow \quad \begin{bmatrix} 2 & -5 & 1 \\ 1 & 2 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

— Coefficient matrix

OR ← Optional lines to indicate

$$\left[\begin{array}{ccc|c} 2 & -5 & 1 & 6 \\ 1 & 2 & -1 & 3 \\ -1 & 1 & 2 & 5 \end{array}\right]$$

— Augmented matrix ← "La matrice augmentée"

Matrix

Capital

The "main diagnal"

$$A = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$ → taille de la matrice
m → nombre de lignes
n → nombre de colunnes
a_{ij} → "indexing"

* Matrice triangulaire superieur — $a_{ij} = 0$ pour tout $i > j$.
"upper triangular"

$$\begin{bmatrix} & * & \cdots & * \\ 0 & & \ddots & \vdots \\ \vdots & \ddots & & * \\ 0 & \cdots & 0 & \end{bmatrix}$$

* Matrice triangulaire inférieure — $a_{ij} = 0$ pour tout $i < j$.

$$\begin{bmatrix} & 0 & \cdots & 0 \\ * & & \ddots & \vdots \\ \vdots & \ddots & & 0 \\ * & \cdots & * & \end{bmatrix}$$

* Matrice diagonal

$$\begin{bmatrix} & 0 & \cdots & 0 \\ 0 & & \ddots & \vdots \\ \vdots & \ddots & & 0 \\ 0 & \cdots & 0 & \end{bmatrix}$$

Elementary Row Operations

1. Swap two rows
2. Multiply row by $c \in \mathbb{R} \setminus \{0\}$
3. Add multiple of a row to another

Doing these on augmented matrix gives us an equivalence system

|| Ex.

$$\begin{bmatrix} 2 & -2 & 6 & 2 \\ -4 & 4 & -10 & 4 \\ 2 & 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 1 \\ -4 & 4 & -10 & 4 \\ 2 & 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 0 & 2 & 8 \\ 2 & 0 & 1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 0 & 2 & 8 \\ 0 & 2 & -5 & -4 \end{bmatrix}$$

$$\downarrow$$

Now can solve for z, then y, then x. $\leftarrow$ $\begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 2 & -5 & -4 \\ 0 & 0 & 2 & 8 \end{bmatrix}$