Lec 3

Ex.

Augmented Matrix

$$\begin{bmatrix} 1 & -2 & -2 & 3 & 1 \\ -2 & -4 & 4 & 2 & 6 \\ 0 & 0 & 4 & 8 & 6 \end{bmatrix}$$

$\downarrow \cdots$

Good enough for backsolving from bottom

$$\begin{bmatrix} 1 & -2 & -2 & 3 & 1 \\ 0 & 0 & 4 & 8 & 6 \\ 0 & 0 & 0 & 8 & 8 \end{bmatrix}$$

* ~~Row echelon form~~! ← Not unique, but position of leading entries is unique.

- all zero rows at bottom
- leading entries to left of non-zeros below it.

$x_1 \quad x_2 \quad x_3 \quad x_4$

Leading variables ← solve for these

Free variable ← in terms of these if applicable

$\downarrow \cdots$

$$\begin{bmatrix} 1 & -2 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

* Reduced row echelon form

- Leading entries all 1s
- All 0s above leading entries

$\downarrow$

$x_1 = -3 - 2x_2$
$x_3 = -\frac{1}{2}$
$x_4 = 1$

Solution

Vector Spaces

Let $\mathbb{R}^n = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$, $x_i \in \mathbb{R}$ for $i = 1, \cdots, n$

- Vector addition
- Scaler multiplication

* Linear combination — $\underbrace{c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k v_k}_{\text{finite expression}}$ with $\vec{v}_i \in \mathbb{R}^n$ and $c_i \in \mathbb{R}$