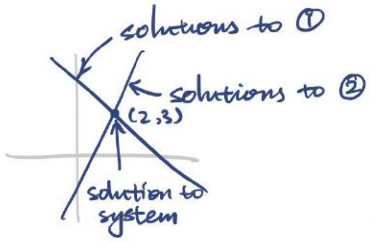
Lec 5

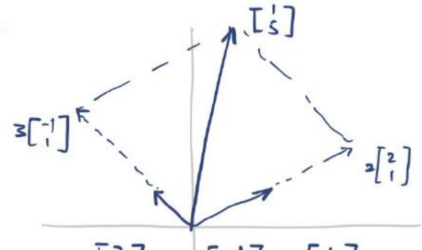
* Connecting systems with n variables and $\mathbb{R}^n$

Ex. $2x - y = 1$ ① Suppose $(x, y) = (\alpha, \beta)$ is a solution.
$x + y = 5$ ② Then $\langle \alpha, \beta \rangle$ can be a vector in $\mathbb{R}^2$.



"Row picture" interpretation

"Column picture". Finding linear combination



$$x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

In $\mathbb{R}^3$

Intersection between 3 planes ← Row picture

Linear combination ← Column picture

$$x\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + y\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + z\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Dot products?

Normal vectors to those planes

$$n_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad n_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \quad n_3 = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} \quad v = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \begin{aligned} n_1 \cdot v &= d_1 \\ n_2 \cdot v &= d_2 \\ n_3 \cdot v &= d_3 \end{aligned}$$

"Normal form"

Generic case

$$\begin{aligned} a_{11}x_1 + \cdots + a_{1m}x_m &= b_1 \\ &\vdots \\ a_{n1}x_1 + \cdots + a_{nm}x_m &= b_n \end{aligned}$$

— Row picture is in $\mathbb{R}^m$

If $n = m$, we say the system is in "standard form"

Column picture is in $\mathbb{R}^n$

Vector form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 2 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -8 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 3 \\ -2 \\ 1 \end{bmatrix}$$

← Kind of already a solution.

Solving a system could be changing from standard form to vector form.

Dot product properties

Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$, $c \in \mathbb{R}$, then

1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
3. $(c\vec{u}) \cdot \vec{v} = c(\vec{u} \cdot \vec{v})$
4. $\vec{u} \cdot \vec{u} \geq 0$