Lec 7

* Alt definition for L.T.

$T: V \to W$ is L.T. iff

$\forall v_1, \dots, v_k \in V,\ c_1, \dots, c_k \in \mathbb{R},\ T(c_1v_1 + \dots + c_kv_k) = c_1T(v_1) + \dots + c_kT(v_k)$

* Matrix for L.T.

Ex. $T: \mathbb{R}^2 \to \mathbb{R}^3$ is a LT and

$T(e_1) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \qquad T(e_2) = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

Then $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Matrix multiplication.

$$Av = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = v_1 \begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} + \cdots + v_n \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

Sometimes we think of each colume as a vector.

Matrix properties

Let A be matrix, $u, v \in \mathbb{R}^n$, $c \in \mathbb{R}$. Then

1. $A(u+v) = Au + Av$
2. $A(cu) = cAu$

So — matrix multiplications are L.T.s. Namely $T(x) = Ax$, $T: \mathbb{R}^n \to \mathbb{R}^m$
there's bijection between L.T. and the matrix multiplication

Some special L.T.

For any vector spacial V:

- The zero transformation $T_0: V \to V$ via $T_0(v) = \vec{0}_v$
- Identity transformation $I: V \to V$ via $T_0(v) = v$.

L.T. compositions.

Let $T: \mathbb{R}^n \to \mathbb{R}^m$, $S: \mathbb{R}^m \to \mathbb{R}^p$

Then $(S \circ T)(u) = B(Au)$