Lec 8

Matrix-vector multiplication [omitted]

| Notice we can have 1×1 matrix $[a]$. Sometimes we blur the line between $[a]$ and a.

Entry indexing

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = \big[a_{ij}\big]_{i,j}$$ ← row i and col j

Then $\rightarrow$ $A^T = \big[a_{ji}\big]_{i,j}$

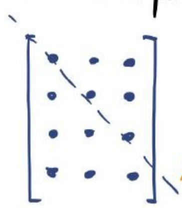
Grabbing row $\rightarrow$ $r_i = \begin{bmatrix} a_{i1} \\ \vdots \\ a_{in} \end{bmatrix}$ ← For this class, we try to keep vectors column vectors.

$\rightarrow$ $r_i^T = [a_{i1} \ \cdots \ a_{in}]$

Grabbing col $\rightarrow$ well... we can grab row of A^T.

Transpose ← write A^T

To transpose:



← Mirror along this axis

Compose

Let $T: \mathbb{R}^n \to \mathbb{R}^m$ via $T(x) = Ax$, $S: \mathbb{R}^m \to \mathbb{R}^p$ via $S(x) = Bx$

$S\circ T: \mathbb{R}^n \to \mathbb{R}^p$ is L.T. $\Rightarrow$ there's unique $p\times n$ matrix C s.t. $(S\circ T)(x) = Cx$.

Well then $C = BA$

Mean while, C must take the form $\begin{bmatrix} | & | & & | \\ (S\circ T)(e_1) & (S\circ T)(e_2) & \cdots & (S\circ T)(e_n) \\ | & | & & | \end{bmatrix}$

Take the j th column. So

$$(S\circ T)(e_j) = A(B(e_j)) = A\left(\begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mn} \end{bmatrix}\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}\right) = A\left(\begin{bmatrix} b_{1j} \\ \vdots \\ b_{mj} \end{bmatrix}\right)$$

$$= b_{1j}\begin{bmatrix} a_{11} \\ \vdots \\ a_{1m} \end{bmatrix} + \cdots + b_{mj}\begin{bmatrix} a_{p1} \\ \vdots \\ a_{pm} \end{bmatrix} = \begin{bmatrix} a_{11}b_{1j} + \cdots + a_{1m}b_{mj} \\ \vdots \\ a_{p1}b_{1j} + \cdots + a_{pm}b_{mj} \end{bmatrix}$$

$$\Rightarrow c_{ij} = \sum_{k=1}^{m} a_{ik}b_{kj} \quad \text{i.e.} \quad C = \left[\sum_{k=1}^{m} a_{ik}b_{kj}\right]_{i,j}$$