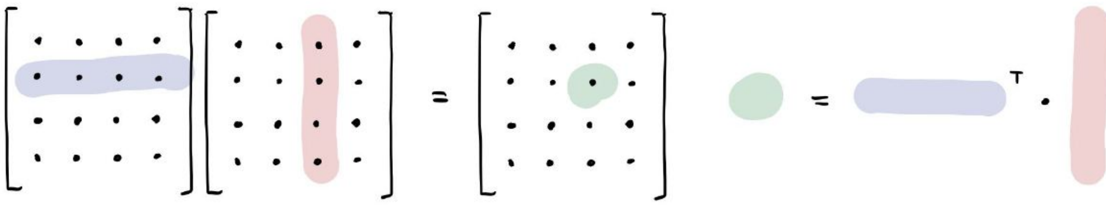
Lec 9

- Midterm next Wed!
- Picking up from matrix multiplication

Another way to think about matrix mul

$$AB = \overset{m\times p}{\left[a_{ij} \right]_{i,j}} \overset{p\times n}{\left[b_{ij} \right]_{i,j}} = \left[\sum_{k=1}^{p} a_{ik} b_{kj} \right]_{ij}$$



$$\Rightarrow AB = \begin{bmatrix} - & r_1^T & - \\ & \vdots & \\ - & r_m^T & - \end{bmatrix} \begin{bmatrix} | & & | \\ c_1 & \cdots & c_n \\ | & & | \end{bmatrix} = \left[r_i \cdot c_j \right]_{i,j}$$

Identity matrix

I_n is an $n\times n$ matrix that looks like:

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

then $AI_n = A$ and $I_nB = B$ provided A, B have right dim for mul.

Matrix addition & scaler mul

$$A + B = \overset{m\times n}{\left[a_{ij} \right]_{i,j}} + \overset{m\times n}{\left[b_{ij} \right]_{i,j}} = \overset{m\times n}{\left[a_{ij} + b_{ij} \right]_{i,j}}$$

$$cA = c \overset{m\times n}{\left[a_{ij} \right]_{i,j}} = \overset{m\times n}{\left[c a_{ij} \right]_{i,j}}$$

Zero matrix

$$\mathbf{0}_{n\times m} = \begin{array}{c} n\times m \\ \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Algebra with Matrix

Let A, B, C be $m\times n$ matrix and $c, d \in \mathbb{R}$.

1. $B + A = A + B$
2. $(A+B)+C = A+(B+C)$
3. $A + \mathbf{0}_{m\times n} = A$
4. $\exists -A_{m\times n}$, $A + (-A) = \mathbf{0}_{m\times n}$
5. $c(A+B) = cA + cB$
6. $(c+d)A = cA + dA$
7. $1A = A$

→ Hmm this looks a lot like vector axioms... what if we say $\mathbb{R}^{m\times n}$ is a vector space of all $m\times n$ matrices

Meanwhile, matrix muls: ← assume ~~type check~~ dim check passes.

1. $(AB)C = A(BC)$
2. $A(B+C) = AB + AC$
3. $(A+B)C = AC + BC$
4. $k(AB) = (kA)B$
5. $I_m A_{m\times n} = A = A_{m\times n} I_n$

Notice that:

1. $A(B+C) = AB + AC$
2. $k(AB) = (kA)B = A(kB)$

So matrix transformation is L.T.