Lec 10

Recall some props of matrix mul

1. $A(B+C) = AB+AC$
2. $k(AB) = (kA)B = A(kB)$

Proof [omitted]. simili for the other props

Transpose properties

1. $(A+B)^T = A^T + B^T$
2. $(kA)^T = k(A^T)$
3. $(AB)^T = B^T A^T$
4. $(A^n)^T = (A^T)^n$ ← induction!

Matrix power

$$A^n = \underbrace{AAA\cdots A}_{n \text{ times}}$$

$$(AB)^T = \left(\overset{m\times n}{\begin{bmatrix} - & r_1^T & - \\ & \vdots & \\ - & r_m^T & - \end{bmatrix}} \overset{n\times p}{\begin{bmatrix} | & & | \\ c_1 & \cdots & c_p \\ | & & | \end{bmatrix}} \right)^T$$

$$= \left(\left[r_j \cdot c_i \right]_{i,j} \right)^T$$

$$= \left[c_i \cdot r_j \right]_{j,i}$$

$$= \overset{p\times n}{\begin{bmatrix} - & c_1^T & - \\ & \vdots & \\ - & c_p^T & - \end{bmatrix}} \overset{n\times m}{\begin{bmatrix} | & & | \\ r_1 & \cdots & r_m \\ | & & | \end{bmatrix}} = B^T A^T$$

Representations of matrix mul

* Def:

$$AB = [a_{ij}]_{i,j} [b_{ij}]_{i,j} = \left[\sum_{l=1}^{n} a_{il} b_{lj} \right]_{i,j}$$

* Row - Col Rep

$$AB = \begin{bmatrix} - \; r_1^T \; - \\ \vdots \\ - \; r_m^T \; - \end{bmatrix} \begin{bmatrix} | & & | \\ c_1 & \cdots & c_p \\ | & & | \end{bmatrix} = \left[\, r_j \cdot c_i \,\right]_{i,j}$$

* Matrix - Col Rep

Every j^{th} col of AB looks like:

$$\begin{bmatrix} - \; r_1^T \; - \\ \vdots \\ - \; r_m^T \; - \end{bmatrix} c_j$$

Whole thing:

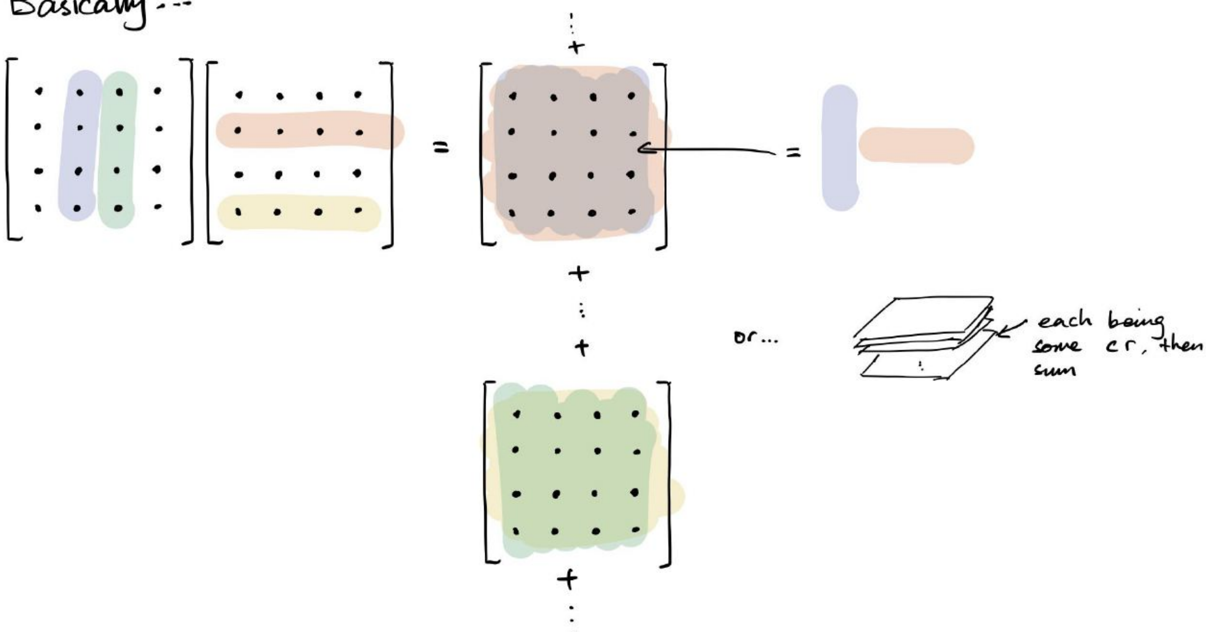
$$AB = \begin{bmatrix} | & & | \\ Ac_1 & \cdots & Ac_p \\ | & & | \end{bmatrix}$$

Can think of as L.T to each col

* Col-Row Rep

$$AB = \left[\sum_{l=1}^{n} a_{il} b_{lj} \right]_{i,j} = \sum_{l=1}^{n} \left[a_{il} b_{lj} \right]_{i,j}$$

Basically...



* Row - Matrix Rep

$$AB = \begin{bmatrix} - & r_1^T & - \\ & \vdots & \\ - & r_m^T & - \end{bmatrix} B = \begin{bmatrix} - & r_1^T B & - \\ & \vdots & \\ - & r_m^T B & - \end{bmatrix}$$

\# Termilogy

Consider $v, w \in \mathbb{R}^n$. We know $v \cdot w \in \mathbb{R}$

These we call "inner product"

1x1!

Also $A_{1 \times n} B_{n \times 1} = [x]$

$A_{n \times 1} B_{1 \times n} = C_{n \times n}$ ← we call that "outer product"