$\boxed{\text{Lec 11}}$

Exam next Wed. Look on Canvas for resources
Review session Mon on Zoom

# Matrix and identity matrix mul.

Recall $$AI_n = A\overbrace{\begin{bmatrix} 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}}^{n\times n} = A\begin{bmatrix} | & & | \\ e_1 & \cdots & e_n \\ | & & | \end{bmatrix} = \begin{bmatrix} | & & | \\ Ae_1 & \cdots & Ae_n \\ | & & | \end{bmatrix}$$

But notice $Ae_j$ is just $j^{th}$ col of $A$, so $AI_n = A$

Also $I_n B_{n\times p} = B_{n\times p}$

# Matrix Inversion

Def: if $A$ is $n\times n$, then its inverse $A^{-1}$ is $n\times n$ s.t. $AA^{-1} = I_n = A^{-1}A$

Ex. $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$. So $AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ indeed.

Theorem: inverse of an invertable matrix is unique.

<u>Proof</u> Let $A', A''$ be inverse of $A$. Then

$$A'' = I_n A'' = (A'A)A'' = A'(AA'') = A' I_n = A'$$

# Properties of inverses

Let $A_{n\times n}$, $B_{n\times n}$ s.t. $\exists A^{-1}, B^{-1}$, let $c \in \mathbb{R}\setminus\{0\}$. Then:

1. $A^{-1}$ invertible and $(A^{-1})^{-1} = A$
2. $cA$ invertible and $(cA)^{-1} = (\frac{1}{c})A^{-1}$
3. $AB$ invertible and $(AB)^{-1} = B^{-1}A^{-1}$
4. $A^T$ invertible and $(A^T)^{-1} = (A^{-1})^T$
5. $\forall p \in \mathbb{Z}^+$, $A^p$ invertible and $(A^p)^{-1} = (A^{-1})^p$

Showing 3: $(AB)(B^{-1}A^{-1}) = AI_nA^{-1} = AA^{-1} = I_n$.
$(B^{-1}A^{-1})(AB)$ simili

# Using inverse to solve sys of lin eq.s

If $A_{n\times n}$ invertible, for all $b \in \mathbb{R}^n$, the system $Ax=b$ has unique solution $x = A^{-1}b$.