Lec 13

Ex $A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$ $\quad \left[A \vdots I_2 \right] = \begin{bmatrix} 2 & 5 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 4 & 0 & 1 \\ 0 & 1 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{4}{3} & -\frac{5}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

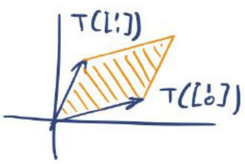
$\Rightarrow A^{-1} = \begin{bmatrix} \frac{4}{3} & -\frac{5}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

Determinants (of square matrix)

→ Determines if inverse exists
→ Tells us some other stuff

Let $A_{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$ ← Measures (signed) area of ▱ ; is zero if L.T. squeezes everything onto a line

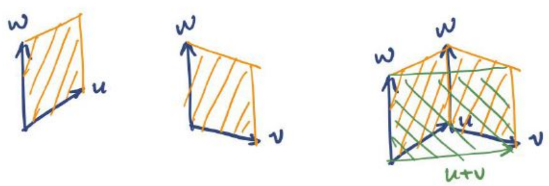


Axioms of determinant.

1. $\det [I_n] = 1$
2. swap 2 row of matrix ⇒ det changes sign
3. det $A_{n\times n}$ is a linear function of first row of A

We were thinking about area. But there might be other things that satisfy these props

Actually works for all rows.
Think: area is linear func of row when other rows are fixed.



Generalise 3 to every row [see proposition 5.5]

Proposition 4 : If two rows of A is zero , then det A = 0

<u>Proof</u>: interchanging those two rows doesn't change matrix.
By 2, $\det A = -\det A$. then $\det A = 0$.

generalises to other rows too

Proposition 5: $1 \mapsto 1 + \lambda i$ for some i that's not first row doesn't change det.

$$\det \begin{bmatrix} (r_1 + t r_i)^T \\ r_2^T \\ \vdots \\ r_n^T \end{bmatrix} = \det \begin{bmatrix} r_1^T \\ r_2^T \\ \vdots \\ r_n^T \end{bmatrix} + t \det \begin{bmatrix} r_i^T \\ r_2^T \\ \vdots \\ r_n^T \end{bmatrix}$$

repeated rows

$$= \det \begin{bmatrix} r_1^T \\ r_2^T \\ \vdots \\ r_n^T \end{bmatrix} + 0$$

Original matrix