$\boxed{\text{Lec 17}}$

Recall: Let $A_{n\times n}$, let A_{ij} be A with i^{th} col j^{th} row removed. Then:

$$\det A = \sum_{j=1}^{n} (-1)^{j+1} a_{1j} \det A_{1j}$$

Ex. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

$= a(ei - fh) - b(di - fg) + c(dh - eg)$

$=$ whatever

Expanding across other rows

<u>Thm</u>. we can do it on any row

Pf. move i^{th} row to top, retain order requires $(i-1)$ row swaps

then $\det A = (-1)^{i-1} \sum_{j=1}^{n} (-1)^{j+1} a_{ij} \det A_{ij}$

$$\Rightarrow \quad \det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$$

* Trick to remember:

$$\left[(-1)^{i+j}\right] \to \begin{bmatrix} + & - & + & \cdots \\ - & + & - & \\ + & - & + & \\ \vdots & & & \ddots \end{bmatrix}$$

<u>Thm</u>. we can do columns too!

$$\det A = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$$

<u>Defs</u>. A_{ij} is the <u>ij-minor</u> of A

$c_{ij} = (-1)^{i+j} \det A_{ij}$ is the ij-cofactor

← easy col

Ex. $\begin{vmatrix} 2 & -3 & 0 & 1 \\ 5 & 4 & 2 & 0 \\ 1 & -1 & 0 & 3 \\ -2 & 1 & 0 & 0 \end{vmatrix} = -2\begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 3 \\ -2 & 1 & 0 \end{vmatrix} = -2\left(-2\begin{vmatrix} -3 & 1 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}\right)$

$= -2(-2(-8) - (5))$

$= -22$

Induction [omitted]