Lec 19

Recall the eigenvec equation

$$A\vec{x} = \lambda \vec{x} - \neq \vec{0}$$

e-vec e-val

More eigen theorems

Thm: Let $A_{n\times n}$ have e-vals $\lambda_1, \dots, \lambda_k$ w/ corresponding e-vecs $v_1, \dots, v_k$.
if x is linear combo

$$x = c_1 v_1 + \cdots + c_k v_k$$

then

$$A^m x = c_1 \lambda_1^m v_1 + \cdots + c_k \lambda_k^m v_k$$

Pf $\quad A^m(c_1 v_1 + \cdots + c_k v_k) = c_1 A^m v_1 + \cdots$
$\qquad\qquad = c_1 \lambda_1^m v_1 + \cdots \quad \square$

Computation applications – huge power of matrix

Consider: $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ w/ $\lambda_1 = 2,\ v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$; $\lambda_2 = 4,\ v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

* What if we want to find A^{30}?

Notice : $A^{30} = \begin{bmatrix} A^{30}\begin{bmatrix} 1 \\ 0 \end{bmatrix} & A^{30}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{2}(v_2 - v_1)$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{2}(v_1 + v_2)$

$$\text{so } A^{30}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{2}A^{30}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{2}A^{30}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}4^{30}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{2}2^{30}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

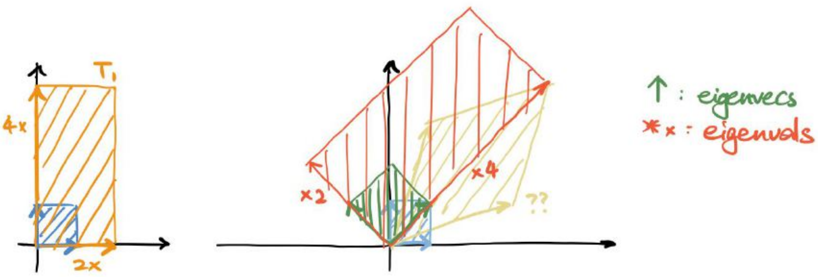
$$= \begin{bmatrix} 2^{59} + 2^{29} \\ 2^{59} - 2^{29} \end{bmatrix}$$

$$\text{and } A^{30}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{2}A^{30}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{2}A^{30}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\vdots$

Eigen* and L.T.

Consider $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ via $x \mapsto \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} x$, T_2 via $x \mapsto \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} x$



Consider :

$A = \begin{bmatrix} 7 & 1 & -2 \\ -3 & 3 & 6 \\ 2 & 2 & 2 \end{bmatrix}$

WTS $\lambda = 6$ is a e-val $\longrightarrow$ show $\det(A - 6I) = 0$

WTF all corresponding e-vecs $\longrightarrow$ solve $(A - 6I)\vec{x} = \vec{0}$ for non-zero $\vec{x}$

↳ $x = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

Notice : $\lambda = 6$ is double root, and corresponding eigenvecs has 2 free vars

* algebraic multiplicity is the num of times λ appear as root .