Lec 30

More properties of orth. matrix Q

<u>Thm</u> Given orth. matrix $Q_{n\times n}$, $R_{n\times n}$, then

1. rows of Q are orthonormal
2. Q^{-1} is orthogonal
3. $\det Q = \pm 1$
4. Eval λ of Q is ± 1
5. QR is orthogonal

<u>Proof</u>

(4) $Qv = \lambda v \Rightarrow \|Qv\| = \|\lambda v\| \Rightarrow \|v\| = |\lambda|\|v\| \Rightarrow |\lambda| = 1$

(5) [think preserving stuff twice]

Finding orth basis — Gram-Schmidt Process

Say we have basis $\{x_1, x_2, x_3\}$ in $\mathbb{R}^3$, find orth. basis $\{v_1, v_2, v_3\}$.

Want:

- $\text{span}(v_1) = \text{span}(x_1)$
- $\text{span}(v_1, v_2) = \text{span}(x_1, x_2)$
- $\text{span}(v_1, v_2, x_3) = \text{span}(x_1, x_2, x_3)$

(⋮)

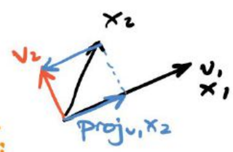
Can generalise easily

Let: $v_1 = x_1$

$v_2 = x_2 - \text{proj}_{v_1} x_2$

$v_3 = x_3 - \underbrace{\text{proj}_{v_1} x_3 - \text{proj}_{v_2} x_3}_{\text{viz. } \text{proj}_{\text{span}(v_1, v_2)} x_3}$

(⋮)



⋮

[generalisation left as brain exercise]

[example omitted]

[span requirement proof omitted]

Then, normalise all v_i to get orthonormal basis :

return $\left\{ \dfrac{v_1}{\|v_1\|}, \dfrac{v_2}{\|v_2\|}, \dfrac{v_3}{\|v_3\|} \ (, \ldots) \right\}$

* textbook calls this modified Gram-Schmidt

* Note: this also generalises to any I.P.S.

[example omitted]