21-259

Calculus in Three Dimensions

Spring 2023

At Carnegie Mellon University

Notes by Lómenoirë Mortecc.

Lec 1

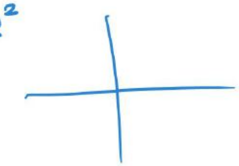
- Syllabus
- Homework
- Late:
  24h — 5% off
  48h — 10% off
- Textbook (TB)
  OpenStax Calculus Volume 3

- Grading
  30% homework
  15% midterm × 3
  25% final

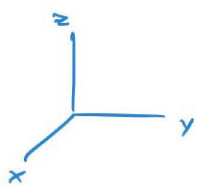
Parametric Equation (TB 1.1 - 1.2)

Vectors in $\mathbb{R}^d$

$\mathbb{R}^2$

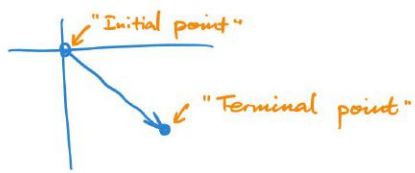


$\mathbb{R}^3$



* Def: a vector in $\mathbb{R}^d$ is a tuple $\langle x_1, x_2, \dots, x_d \rangle$ of $\mathbb{R}$s encoding magnitude and direction.

Ex. $\langle 1, -1 \rangle$



Ex. Initial $(1, 0, 2)$
Terminal $(-2, 1, 3)$

Vec : $\langle -3, 1, 1 \rangle$

Vector operations

* Vector addition :

Given vectors $\vec{v} = \langle x_1, x_2, \dots, x_d \rangle$

$\vec{u} = \langle y_1, y_2, \dots, y_d \rangle$,

their sum $\vec{v} + \vec{u} = \langle x_1 + y_1, x_2 + y_2, \dots, x_d + y_d \rangle$

* Geometric interpretation :

Works in any dimension

parallelogram

* Scaler multiplication

Given vector $\vec{v} = \langle x_1, x_2, \dots, x_d \rangle \in \mathbb{R}^d$

constant $c \in \mathbb{R}$

Then $c\vec{v} = \langle cx_1, cx_2, \dots, cx_d \rangle$

Ex. $\vec{u} := \langle 1, 4, -1 \rangle$, $\vec{v} := \langle 3, 0, 2 \rangle$

$5\vec{v} - \vec{u} =$ [...] $= \langle 14, -4, 11 \rangle$

Magnitude

Given vector $\vec{v} = \langle x_1, \dots, x_d \rangle$

The magnitude or length of $\vec{v}$ is defined as:

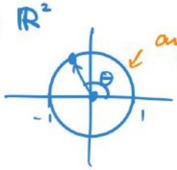
$$\|\vec{v}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_d^2}$$

← Comes from pythagorean (?) theorem repeatedly applying to "reduce" dims

Unit vectors and direction

A unit vector in $\mathbb{R}^d$ is a vector $\vec{u}$ with $\|\vec{u}\| = 1$

In $\mathbb{R}^2$



anything with distance 1 can be a unit vector.

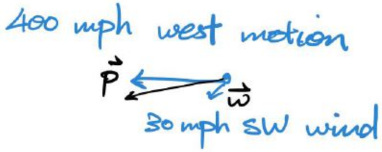
Note that angle in $\mathbb{R}^2$ works too.
Since we can use $\langle \cos\theta, \sin\theta \rangle$ to get the point.

Standard basis vectors in $\mathbb{R}^3$:

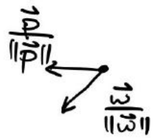
$\vec{i} = \langle 1, 0, 0 \rangle$
$\vec{j} = \langle 0, 1, 0 \rangle$
$\vec{k} = \langle 0, 0, 1 \rangle$

Ex

Suppose



Directions :



So:

$$\vec{p} = 400 \langle -1, 0 \rangle$$

$$\vec{w} = 30 \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$$

Resulting vector :

$$\vec{p} + \vec{w} = \left\langle -400 - \frac{30\sqrt{2}}{2}, \frac{-30\sqrt{2}}{2} \right\rangle$$

Resulting speed :

$$\| \vec{p} + \vec{w} \| = [\ldots] \approx 421.7$$

Angle :

$\arctan(\ \ldots\ ) \approx 28^\circ$ south of west