$\boxed{\text{Lec 2}}$

Standard basis vecs for $\mathbb{R}^d$ with $d > 3$.

$e_1 = \langle 1, 0, \cdots, 0 \rangle$

$e_i = \langle 0, \dots, 1, \dots, 0 \rangle$

$\uparrow$ ith coordinate

Dot Product

aka "inner product"

Def: Let $\vec{u} = \langle x_1, \dots, x_d \rangle$

$\vec{v} = \langle y_1, \dots, y_d \rangle$

Then $\vec{u} \cdot \vec{v} = \sum_{i=1}^{d} x_i y_i$

A scaler, sum of product for every pair of coord in same dim.

Some properties

- $c(\vec{u} \cdot \vec{v}) = (c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v})$
- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $\vec{u} + (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u}\vec{w}$
- $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$

← Proofs follow from definition

Dot prod applications

* Measuring angles

Let $\vec{u}, \vec{v} \in \mathbb{R}^d$

Let θ:

Theorem "Physicists' dot product"

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$\vec{u} = \langle 0, 3, -3 \rangle$
$\vec{v} = \langle 2, 1, -1 \rangle$

Find angle between them.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

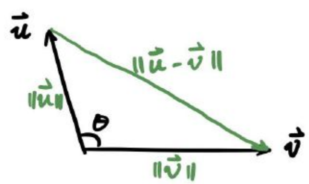
$$\Rightarrow \theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}\right)$$

$$= \cos^{-1}\left(\frac{0+3+3}{\sqrt{6} \cdot \sqrt{18}}\right)$$

$$= \cos^{-1}\left(\frac{6}{\sqrt{6 \cdot 18}}\right)$$

$$\approx 54.7^\circ$$

Proof:



By law of cos:

$$\|\vec{u} - \vec{v}\|^2 = \|\vec{v}\|^2 + \|u\|^2 - 2\|\vec{v}\| \|\vec{u}\| \cos\theta$$

$$(\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = \vec{v} \cdot \vec{v} + \vec{u} \cdot \vec{u} - 2\|\vec{v}\| \|\vec{u}\| \cos\theta$$

$$\vec{v} \cdot \vec{v} - 2\vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{u} = \vec{v} \cdot \vec{v} + \vec{u} \cdot \vec{u} - 2\|\vec{v}\| \|\vec{u}\| \cos\theta$$

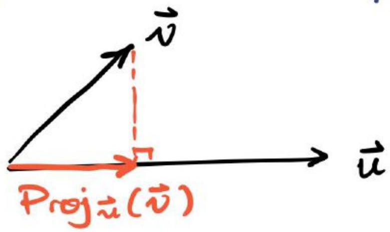
$$\vec{u} \cdot \vec{v} = \|\vec{v}\| \|\vec{u}\| \cos\theta$$

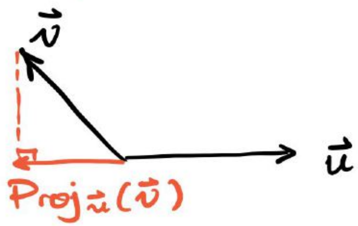
* Vector projection

Let $\vec{u}, \vec{v} \in \mathbb{R}^d$

$\text{Proj}_{\vec{u}}(\vec{v})$ measures portion of $\vec{v}$ in direction of $\vec{u}$

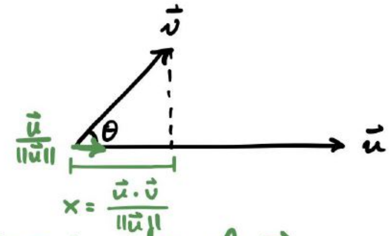
Projection of $\vec{v}$ onto $\vec{u}$





Formula:

$$\text{Proj}_{\vec{u}}(\vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\,\vec{u}$$



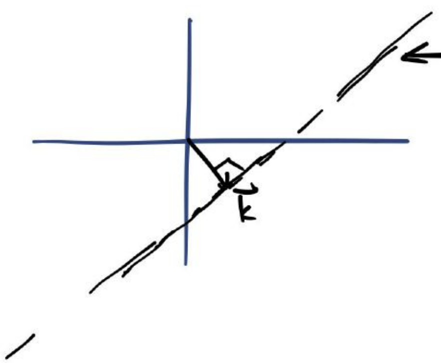
Well, $\frac{\vec{u}}{\|\vec{u}\|}$ gives unit vector in dir of $\vec{u}$

Also $\cos\theta = \frac{x}{\|\vec{v}\|}$

So $\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{x}{\|\vec{v}\|} \Rightarrow x = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|}$

Called scalar projection.
Denoted $\text{comp}_{\vec{u}}(\vec{v})$

In $\mathbb{R}^2$ find set of vectors $\vec{v}$ s.t. $\text{comp}_{\vec{u}}(\vec{v}) = k$ for $\vec{u} \neq 0$ and $\vec{k}$ constant.



This line!

Try do it using formula