Lec 3

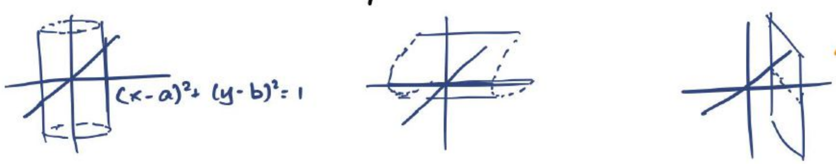
Basic surfaces , cross products, quadric surfaces

Basic Surfaces (in $\mathbb{R}^3$)

- Planes $ax + by + cz = d$ → $ax+by=c$ in $\mathbb{R}^2$
- Spheres $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ → $(x-a)^2 + (y-b)^2 = c$

for fixed a, b, c, d.
r is radius, centre is at (a, b, c)

- Cylinders
  - Right cylinder — solution to equation in less than 3 variables in $\mathbb{R}^3$
    some shape stretched out in a direction



a stretched line

Generalise to any dim

In $\mathbb{R}^d$:

- hyperplane $a_1x_1 + a_2x_2 + \cdots + a_d x_d = b$
- hypersphere $(x_1-a_1)^2 + (x_2-a_2)^2 + \cdots + (x_d-a_d) = r^2$

Determine stuff from points.

- $\mathbb{R}^3$ — 3 points on the same line determine a plane

Well — say we have $ax+by+cz=d$.

$\Leftrightarrow \langle a,b,c\rangle \cdot \langle x,y,z\rangle = d$

Suppose $d=0$.

The points are $(0,0,0)$ ← since $d=0$

(x_2, y_2, z_2)

(x_3, y_3, z_3)

↳ Then the plane will be $ax+by+cz=0$.

We want $\langle a,b,c\rangle$ s.t. $\langle a,b,c\rangle \cdot \langle x_2,y_2,z_2\rangle = 0$, $\langle a,b,c\rangle \cdot \langle x_3,y_3,z_3\rangle = 0$ — Hmm that looks like infinite solutions

↳ also called "orthogonal"

What if we find the vector that's perpendicular to $\langle x_2, y_2, z_2\rangle$ and $\langle x_3, y_3, z_3\rangle$

| Recall: $\vec{u}$ and $\vec{v}$ orthogonal if $\vec{u}\cdot\vec{v}=0$

Def: Given $\vec{u} = \langle x_1, y_1, z_1\rangle$

$\vec{v} = \langle x_2, y_2, z_2\rangle$

Their cross product $\vec{u}\times\vec{v}$ is the formal determinant of

$$\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{bmatrix}$$

└ Anticommutative! $\vec{u}\times\vec{v} = -(\vec{v}-\vec{u})$

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad-bc$$

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a\det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b\det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c\det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$\boxed{\begin{aligned}\vec{u}\times\vec{v} &= \vec{i}(y_1z_2 - y_2z_1) - \vec{j}(x_1z_2 - x_2z_1) + \vec{k}(x_1y_2 - x_2y_1) \\ &= \langle y_1z_2 - y_2z_1,\ x_2z_1 - x_1z_2,\ x_1y_2 - x_2y_1\rangle\end{aligned}}$$

→ Proof is just algebra

Ex. $\langle 5,0,4\rangle \times \langle 2,3,3\rangle$

$$= \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 0 & 4 \\ 2 & 3 & 3 \end{bmatrix} = \langle 0\cdot3 - 3\cdot4,\ 2\cdot4 - 3\cdot5,\ 5\cdot3 - 2\cdot0\rangle$$