Lec 4

Equations of planes and $\mathbb{R}^3$, general version

Consider a plane containing (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)

Let the plane be $ax+by+cz=d$.
Then $\langle a,b,c\rangle$ is a vector normal to the plane

Given point (x_1,y_1,z_1) and normal vector $\vec{n} = \langle a,b,c\rangle$. This determines a plane $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

So finding this normal vector helps us find the plane.

1. Get two vectors from the three points
2. Cross them

i.e. $\langle a,b,c\rangle = \langle x_2-x_1, y_2-y_1, z_2-z_1\rangle \times \langle x_3-x_1, y_3-y_1, z_3-z_1\rangle$
then $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ is the equation

Higher dim generalisation: normal vector + plane always work!
But — cross product only works in $\mathbb{R}^3$! Use the higher dim equivalence.

In $\mathbb{R}^4$, it would be $\begin{bmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 & \vec{e}_4 \\ - & \vec{v}_2-\vec{v}_1 & - & \\ - & \vec{v}_3-\vec{v}_1 & - & \\ - & \vec{v}_4-\vec{v}_1 & - & \end{bmatrix}$

Lines in $\mathbb{R}^3$

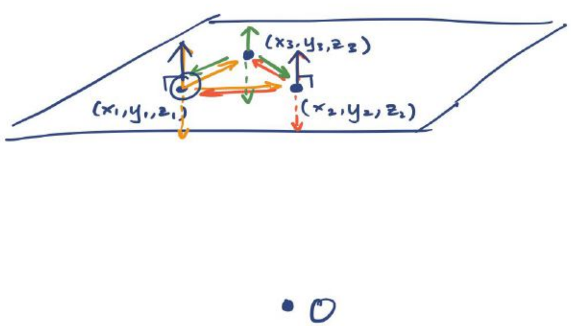
→ Can be determined by a point and a direction vector

→ Can be determined by two points
Let P, Q be points
Then vector equation is $\vec{r}(t) = \underbrace{\overrightarrow{OP}}_{\text{move whatever to P}} + \underbrace{\overrightarrow{PQ}}_{\text{a vector on that line}}\, t$

Ex. $P = (4, -1, 2)$, $Q = (3, -1, -1)$
$\vec{r}(t) = \langle 4, -1, 2 \rangle + t\langle -1, 0, 3 \rangle$ is the vector equation
↳ Parametric form
$x = 4 - t$
$y = t$
$z = 2 - 3t$

Question

Any center point ?



↙ Yes, any center point would work.