$\boxed{\text{Lec 5}}$ Parallel, Intersecting Lines / Planes, Skew Lines, Distances

* Line arrangement in $\mathbb{R}^3$

They are either:

* Parallel
* Intersecting
* Skew (neither) ← Most likely when drawing random lines.
(actually 100% percent probability)

Intersecting

Ex. find intersection between $\vec{r}_1(t) = \langle 2,3,3 \rangle + t\langle 1,1,2 \rangle$
$\vec{r}_2(t) = \langle 1,4,2 \rangle + t\langle 1,-1,1 \rangle$

→ Set up a system (with different variable for each)

So
$$\left.\begin{aligned} 2+t &= 1+s \\ 3+t &= 4-s \\ 3+2t &= 2+s \end{aligned}\right] \text{Then solve it.}$$

$\cdots \Rightarrow t=0,\ s=1.$

So point of intersection is $(2+0, 3+0, 3+0) = (2,3,3)$

Parallel

Def: $\vec{u}$ and $\vec{v}$ with $\vec{u}, \vec{v} \neq \vec{0}$ if $\vec{u} = c\vec{v}$ for $c \neq 0$.
or iff $\vec{u} \times \vec{v} = \vec{0}$.
or iff $|\vec{u} \cdot \vec{v}| = \|\vec{u}\|\|\vec{v}\|$.

Plane arrangements in $\mathbb{R}^3$.

They are either :

* Parallel ← Normal vectors are parallel
* Intersecting ← Intersection is a line

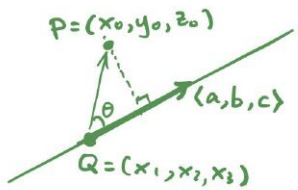
Ex. fine line of intersection between $\left.\begin{array}{l} 5x+3y+z=10 \\ 8x+9y+2z=9 \end{array}\right]$ Just solve it by picking one of x,y,z as free variable.

Distance between point and line in $\mathbb{R}^3$.

Point : $P = (x_0, y_0, z_0)$
Line : $\vec{r}(t) = \langle x_1, y_1, z_1 \rangle + t\langle a, b, c\rangle$

→ Find t such that the distance between point at $\vec{r}(t)$ and P is minimised (shortcut is to minimise squared distance)

→ Geometric way



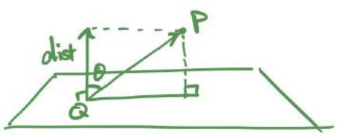
$$\text{dist} = \|\vec{PQ}\| \sin\theta = \|\vec{PQ}\| \frac{\|\vec{PQ} \times \langle a,b,c\rangle\|}{\|\vec{PQ}\|\|\langle a,b,c\rangle\|}$$
$$= \frac{\|\vec{PQ} \times \langle a,b,c\rangle\|}{\|\langle a,b,c\rangle\|}$$

Distance between point and plane in $\mathbb{R}^3$

Let $P = (x_0, y_0, z_0)$
$ax + by + cz = d$

→ We can also minimise

→ Geometric



$$\cos\theta = \frac{\text{dist}}{\|\vec{PQ}\|}$$

$$= \frac{\langle a,b,c\rangle \cdot \vec{QP}}{\|\langle a,b,c\rangle\| \|\vec{QP}\|}$$

$$\text{dist} = \frac{\langle a,b,c\rangle \cdot \vec{QP}}{\|\langle a,b,c\rangle\|}$$