Lec 6 | Quadric Surfaces

→ The $\mathbb{R}^3$ analogue of conic sections

* Conic sections (refresher)

- parabola $x^2 = y + 1$
- hyperbola $\frac{x^2}{5} - \frac{y^2}{11} = 3$
- ellipses $\frac{x^2}{4} + y^2 = 2$
- X shape $x^2 = 3y^2$

* Quadric surfaces

* We get these when solving degree 2 polynomials with 3 vars

- $x^2 + y^2 + z^2 = 1$ → sphere 
- $x^2 + y^2 = z^2$ → cone 

usually we don't deal with these

they typically make things rotate

* General form : $Ax^2 + By^2 + Cz^2 + \underbrace{Dxy + Exz + Fzy}$ $+ Hx + Iy + Jz + K = 0$
with at least one of $A, B, C, D, E, F \neq 0$.

Types

* Cone → 
* Ellipsoid → 
* Hyperboloid with one sheet 
* Hyperboloid with two sheet 

Homogeneous. All to 2nd power

$$x^2 + y^2 = z^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Identifying these :

1. move constants to one side
2. make constant positive
3. see num of − signs.

   0 ⇒ ellipsoid

   1 ⇒ hyperboloid 1 sheet

   2 ⇒ hyperboloid 2 sheet

   3 ⇒ : C ∅

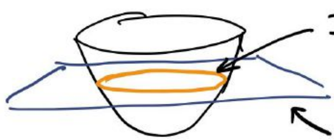
* Elliptic paraboloid →  $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ ← slices U or O /.
* Hyperbolic paraboloid  $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ ← slices in z direction ⌣⌢ / X /)(.
slices in another direction all V

* These can all be sliced from $\mathbb{R}^4$ cones (there are 2 types of 4D cones, we mean at least one of them)

Other situations

* If one of x, y, z absent, we get a cylinder
* If one if to 2nd power and other to 1st power, a slanted cylinder

Traces



Intersection is the trace

plane parallel to one of the axes