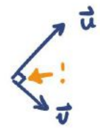
Lec 9

\# Some space curve assumptions.

Vector-based function $\vec{r}(t)$ in $\mathbb{R}^n$ is C^n if $\vec{r}'(t), \vec{r}''(t), \dots, \vec{r}^{(n)}(t)$ all exist and continuous.

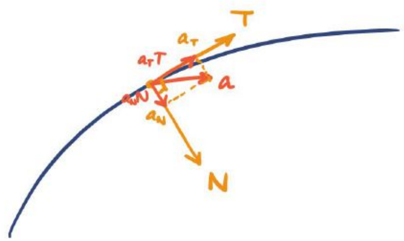
\# Change of basis in $\mathbb{R}^2$

Given:



Then for any $\vec{w} \in \mathbb{R}^2$,
$\exists!\, a, b \in \mathbb{R}$, $\vec{w} = a\vec{u} + b\vec{v}$

$\Rightarrow$ We can have



$$\vec{a}(t) = \underbrace{a_T \vec{T}(t)}_{\text{Acceleration in direction of motion}} + \underbrace{a_N \vec{N}(t)}_{\text{Centripetal acceleration}}$$

Acceleration can always be expressed as linear combo of $T(t)$ and $N(t)$

in which

$a_T = a \cdot T$, $a_N = a \cdot N$

Theorem:

change in speed

$$a_T = s'(t) \quad \text{where } s(t) = \|\vec{v}(t)\|$$

$$a_N = (s(t))^2 K \quad \text{where } K \text{ is curvature}$$

$$= \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|}$$

$$= \sqrt{\|\vec{a}\|^2 - a_T^2}$$

from Pythagorean theorem

$$\vec{N} = \frac{T'(t)}{\|T'(t)\|}$$

$$K = \frac{\|T'(t)\|}{\|\vec{v}(t)\|}$$

$$\vec{v}(t) = \|\vec{v}(t)\| \vec{T}(t)$$

$$= s(t)\, \vec{T}(t)$$

$$\frac{d}{dt}\vec{v}(t) = \frac{d}{dt} s(t)\, \vec{T}(t)$$

$$\underbrace{v'(t)}_{} = s'(t)\vec{T}(t) + s(t)\underbrace{\vec{T}'(t)}_{}$$

$$\underbrace{a(t)}_{} = s'(t)\vec{T}(t) + \underbrace{\|\vec{v}(t)\|\, \|T'(t)\| \vec{N}(t)}_{}$$

$$= \underbrace{s'(t)}_{a_T} \vec{T}(t) + \underbrace{\|\vec{v}(t)\|^2 K}_{a_N} \vec{N}(t)$$