Lec 10 Derivative of function with multi. variable.

Definitions

* **Scalar-valued function**

$f: D \subseteq \mathbb{R}^d \to \mathbb{R}$ assigns each $(x_1, \dots, x_d) \in D$ a real num $z = f(x_1, \dots, x_d)$

eg.

looks like this 

$z = \sqrt{4 - 2x^2 - y^2}$ ← passes vertical line test

* **Traces & level set**

We can think of it as $w = f(x,y) - z$ at $w = 0$.
this is useful for finding tangent plane

Say $z = f(x, y)$.

2D independent var

A <u>trace</u> is the 2D intersection parallel to one coord axis

A <u>level set</u> is the $z = k$ trace of z. In case of $\mathbb{R}^3$ this aka <u>level curve</u>

A <u>verticle trace</u> is a trace along one of independent var.

A <u>contour map</u> is sketch of level curve in $\mathbb{R}^2$ ←

Now consider $w = \sqrt{1 - x^2 - y^2 - z^2}$ ← hemisphere in $\mathbb{R}^4$

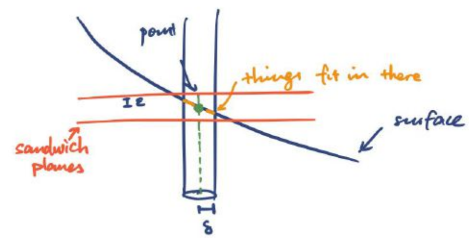
↳ Sphere level surfaces look like spheres for $0 \leq w < 1$
  ↳ Contour map are nested spheres

Limit

$z = f(x,y)$

Def: $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$. For any $\varepsilon > 0$, there exists $\delta > 0$ s.t. if $\text{dist}((x,y),(x_0,y_0)) \in (0,\delta)$, $f(x,y) \in (L-\varepsilon, L+\varepsilon)$

↳ See geogebra demo



To show limit DNE, show the limit is different from different directions (there are infinite possible directions)

To show limit exists and compute:

* Continuity: f is continuous at (x_0,y_0) if $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L_1 \wedge \lim_{(x,y)\to(x_0,y_0)} g(x,y) = L_2 \Rightarrow$

* Suming limit: $\lim_{(x,y)\to(x_0,y_0)} f(x,y) + g(x,y) = L_1 + L_2$

* Product of limit $\lim_{(x,y)\to(x_0,y_0)} f(x,y)\, g(x,y) = L_1 \cdot L_2$

* Quotient of limit $\lim_{(x,y)\to(x_0,y_0)} f(x,y)/g(x,y) = L_1 / L_2$ if $L_2 \neq 0$.