Lec 11

Limit with multiple variables

$$\lim_{(x,y)\to(a,b)} \frac{4x^2+10y+4}{\underbrace{4x^2-10y^2+6}_{\text{notice this goes 0}}} = \frac{2}{3}$$

In case where bottom doesn't go zero, just evaluate.

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{3x^2+y^2}$$ → notice direct eval gets us $\frac{0}{0}$.

Maybe DNE

To show DNE: pick two paths to approach and they approach different val

- On $y=0$: $\cdots = \lim_{x\to0^-} \frac{2x(0)}{3x^2+0^2} = \lim_{x\to0} \frac{0}{3x^2} = 0$
- On $y=x$: $\cdots = \lim_{x\to0^+} \frac{2xx}{3x^2+x^2} = \lim_{x\to0^+} \frac{2x^2}{4x^2} = \frac{1}{2}$

different when approaching from different dir

$y=x$

$y=0$

Partial Derivative

Let $z = f(x,y)$

* Partial derivative w.r.t. x is $\frac{\partial f}{\partial x}$ or $\frac{\partial z}{\partial x}$ or f_x def by

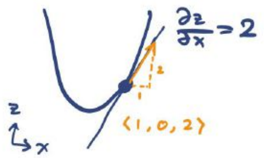
$$\frac{\partial f}{\partial x} = \lim_{h\to0} \frac{f(x+h,y)-f(x,y)}{h}$$ at points where the lim exists

i.e. fixing y and differentiating

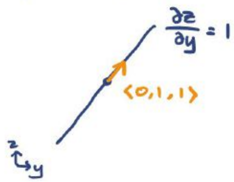
Tangent plane

Find tangent plane to $z = x^2 - 2x + xy + 1$ at $(1, 2)$

plane $y = 2$



plane $x = 1$



That gives us a point and two vectors.
Then we can just:
1. Cross to get normal vector
2. Shift plane to contain points

General formula: $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$ ← (If it exists)

Gradient

Let $z = f(x, y)$

The gradient $\nabla f = \langle f_x, f_y \rangle$ ← Extends to more variables

Theorem: if $h = f(x, y, z)$ is a level surface, the tangent plane at (a, b, c) is

$$\nabla f \cdot \langle x-a, y-b, z-c \rangle = 0$$

Ex: say $z = f(x, y) \Rightarrow 0 = f(x, y) - z$ ← that's a level surface

$$\frac{\partial (f(x,y) - z)}{\partial x} = f_x(x, y)$$

$$\frac{\partial (f(x,y) - z)}{\partial y} = f_y(x, y)$$

$$\frac{\partial (f(x,y) - z)}{\partial z} = -1$$

Then tangent plane at $(a, b, f(a,b))$ is

$$\langle f_x, f_y, -1 \rangle \cdot \langle x-a, y-b, z-f(a,b) \rangle = 0$$