$$\boxed{\text{Lec 13}}$$

# Multivari chain rule

2D: $y = e^{-x^2}$ $\quad$ $y = e^{u}$, $u = -x^2$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

3D: $\overbrace{z = f(x,y),\ x = h(t),\ y = h(t)}^{*}$. Find $\frac{dz}{dt}$

Suppose 

, $\frac{dw}{dt} = \frac{1}{2}$, $\frac{dl}{dt} = -\frac{1}{5}$. $\frac{dA}{dt} = ?$

Theorem: if $*$, $\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}}$ ← Chain rule for more than 1 var. Generalises to higher dim.

Notation: $\frac{dz}{dx}$ if $z$ only depends on $x$

$\frac{\partial z}{\partial x}$ if $z$ depends on $x$ and sth else

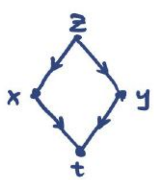
In ex: $A = wl$

$$\frac{dA}{dt} = \frac{\partial A}{\partial l}\frac{dl}{dt} + \frac{\partial A}{\partial w}\frac{dw}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt} = -5\frac{1}{5} + 10\frac{1}{2} = 4$$

# Tree diagram (var dependency)

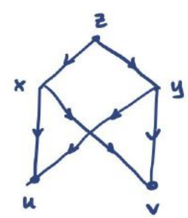
1 Var: 


$$\frac{dy}{du} = \frac{dy}{dx}\frac{dx}{du}$$

Prev ex: 


$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

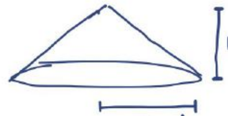
↑ Each path, then add.

Ex. 


$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

⋮

More Ex.



10 $\quad \frac{dh}{dt} = -1$

5 $\quad \frac{dr}{dt} = ?$ $\quad \frac{dV}{dt} = 0$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$

$= \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

$0 = \frac{2}{3}\pi (5)(10)\frac{dr}{dt} + \frac{1}{3}\pi (25)(-1)$

$\frac{25}{3}\pi = \frac{100}{3}\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4}$

# Implicit differentiation

We could also have:
→ linear approx after solve for z
→ use gradient

Find tangent plane to ellipsoid $x^2 + \frac{y^2}{9} + 2z^2 = 1$ at $\left(\frac{3}{\sqrt{28}}, \frac{-3}{\sqrt{28}}, \frac{-3}{\sqrt{28}}\right)$

Implicit diff way:

$\frac{\partial}{\partial x}\left[x^2 + \frac{y^2}{9} + 2z^2\right] = \frac{\partial}{\partial x}[\ 1\ ]$

Since we picked x, y as independent and z as dependent

$2x\frac{\partial x}{\partial x} + \frac{2y}{9}\frac{\partial y}{\partial x} + 4z\frac{\partial z}{\partial x} = 0$ $\qquad \frac{\partial x}{\partial x} = 1,\ \frac{\partial y}{\partial x} = 0$

$2x + 4z\frac{\partial z}{\partial x} = 0$

then solve for $\frac{\partial z}{\partial x}$, do same for $\frac{\partial z}{\partial y}$, then get plane using $L(x,y)$ formula

… $-\frac{3}{\sqrt{28}} + \frac{1}{2}\left(x - \frac{3}{\sqrt{28}}\right) - \frac{1}{18}\left(y + \frac{3}{\sqrt{28}}\right)$

# Tangent plane from gradient

We treat it as level surf $w = x^2 + \frac{y^2}{9} + 2z^2$

$\nabla w = \langle 2x, \frac{2y}{9}, 4z \rangle$. $\nabla w$ at point $= \langle 2\frac{3}{\sqrt{28}}, \frac{-6}{9\sqrt{28}}, -\frac{12}{\sqrt{28}} \rangle$.

Rescale for another normal vec $= \langle 6, -\frac{2}{3}, -12 \rangle$

So a plane: $6\left(x - \frac{3}{\sqrt{28}}\right) - \frac{2}{3}\left(y + \frac{3}{\sqrt{28}}\right) - 12\left(z + \frac{3}{\sqrt{28}}\right) = 0$