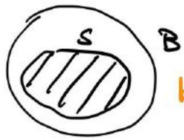
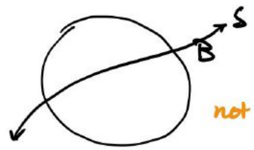
Lec 17

Bounded set

<u>Def</u>: $S \subseteq \mathbb{R}^d$ is bounded if there exists some d-dimensional sphere B with finite radius s.t. $S \subseteq B$



bounded



not bounded

Extreme val thm.

For closed, unbounded $f: S \to \mathbb{R}$, $S \subseteq \mathbb{R}^d$, f continuous, f has abs max and min on S.

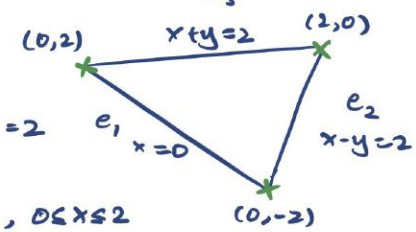
Recall Calc I Extreme Val Thm:
If $f[a,b] \to \mathbb{R}$ cont., f has abs & min on $[a,b]$.
To find:

1. Find crit points in $[a,b]$ (viz. $f'=0$, f' DNE)
2. Check f at a, b, and crit points

In 3D, similar but boundry more complicated — check all border!

Ex. find abs max & min of $f(x,y) = z = x^2+y^2-2x$ on triangle $((2,0),(0,-2),(0,2))$

1. Check vertices — $f(0,2)=\underline{4}$, $f(0,-2)=\underline{4}$, $f(2,0)=\underline{0}$
2. Check along edges — lagrange / substitution



$g(x,y) = x-y=2$ ↑

- e_1 $x=0$
  $z = y^2$
  $f' = 2y = 0$
  $y = 0$
  $f(0,0) = \underline{0}$
- e_2 lagrange: opt $f(x,y) = x^2+y^2-2x$ subject to $x-y=2$, $0\le x\le 2$

$$\begin{matrix}\nabla f = \lambda \nabla g \\ x-y=2\end{matrix} \Rightarrow \begin{matrix} 2x-2=\lambda \\ 2y=-\lambda \\ x-y=2\end{matrix} \cdots\Rightarrow x=\tfrac{3}{2},\ y=-\tfrac{1}{2},\quad f(\tfrac{3}{2},-\tfrac{1}{2}) = \underline{-\tfrac{1}{2}}$$

- e_3 $\cdots \to (\frac{3}{2},\frac{1}{2})$, $f(\frac{3}{2},\frac{1}{2}) = \underline{-\frac{1}{2}}$

3. Check interior
   $\nabla f = \langle 2x-2, 2y\rangle = 0$
   $\Rightarrow (x,y) = (1,0)$, within region
   $f(1,0) = \underline{-1}$

$\Rightarrow$ max $=4$, min $=-1$

Ex. find max, min of $f(x,y) = \dfrac{2x}{x^2+y^2+1}$ in $\{x^2+y^2\le 4\}$.

Along boundry: opt $f(x,y)$ subject to $g(x,y) = x^2+y^2=4$

$$\nabla f = \left\langle \frac{(x^2+y^2+1)2-2x(2x)}{(x^2+y^2+1)^2}, \frac{-2y2x}{(x^2+y^2+1)^2}\right\rangle = \lambda\nabla g = \langle 2x, 2y\rangle$$
$$x^2+y^2=4$$

$\cdots\Rightarrow y=0,\ x=\pm 2.$

[then check those points & check interior]