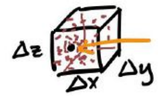
Lec 21

Triple integrals

Def Let f be cont. on closed, bounded $E \subseteq \mathbb{R}^3$,

$$\iiint_E f\,dV = \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} f(x_i^*, y_j^*, z_k^*)\,V_{ijk}$$

$$\approx \sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n} f(x_i^*, y_j^*, z_k^*)\,\Delta x \Delta y \Delta k$$



Ex. Show vol of sphere with radius r

Let E = ball of radius r centered around origin
this is bounded by $\{(x,y,z) \mid x^2+y^2+z^2\}$

$$\iiint_E 1\,dV = \int_{-r}^{r}\int_{-\sqrt{r^2-z^2}}^{\sqrt{r^2-z^2}}\int_{-\sqrt{r^2-z^2-y^2}}^{\sqrt{r^2-z^2-y^2}} 1\,dx\,dy\,dz$$

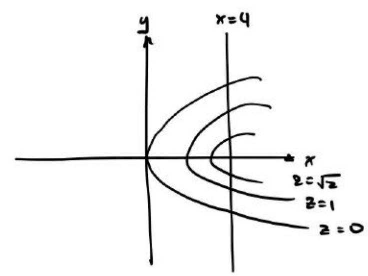
$$= \int_{-r}^{r}\int_{0}^{2\pi}\int_{0}^{\sqrt{r^2-z^2}} s\,ds\,d\theta\,dz$$

$\downarrow$ to cylindrical

$$= \int_{-r}^{r} (r^2-z^2)\pi\,dz$$

Ex. vol enclosed by $x = y^2+z^2$ and plane $x=4$

$$\iiint_E dV = \int_{-2}^{2}\int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}}\int_{y^2+z^2}^{4} 1\,dx\,dy\,dz = \frac{32\pi}{3}$$



min/max of z is when $x=4$, $y=0$
$\Rightarrow -2 \le z \le 2$
fix some z, then min/max of y is $\sqrt{4-z^2} = y \Rightarrow -\sqrt{4-z^2} \le y \le \sqrt{4-z^2}$
fix some y, then min/max of x is $y^2+z^2 \le x \le 4$

Ex. vol of tetrahedron $(0,0,0), (0,1,0), (1,0,0), (0,0,1)$

$$\iiint_E 1 \, dV = \int_0^1 \int_0^{1-z} \int_0^{1-z-y} 1 \, dx \, dy \, dz$$