Lec 25

More centre of mass

Def for random vec $(X_1, \cdots, X_d)$ on D with general density $\rho(x_1, \cdots, x_d)$ the centre of mass is

$$(\bar{x}_1, \cdots, \bar{x}_d) = \left(\frac{\mathbb{E}(X_1)}{m}, \cdots, \frac{\mathbb{E}(X_d)}{m} \right)$$

$$= \left(\frac{\int \cdots \int_D x_1 \rho \, dV}{m}, \cdots, \frac{\int \cdots \int_D x_d \rho \, dV}{m} \right)$$

where $m = \int \cdots \int_D \rho \, dV$

Def for a random var X_1, the k^{th} moment is $\mathbb{E}(X^k)$

Ex. find center of mass for $R = \{(x,y) \mid 9x^2 + y^2 \le 1 \text{ in quadrant } 1\}$, $\rho = \sqrt{9x^2+y^2}$

$$m = \iint_R \sqrt{9x^2+y^2} \, dA = \iint_S \sqrt{u^2+v^2} \cdot \frac{1}{3} \, dA = \iint_T r \cdot \frac{1}{3} \cdot r \, dA$$

$u = 3x \quad v = y$

$x = \frac{u}{3} \quad y = v$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \frac{r^2}{3} \, dr \, d\theta = \frac{\pi}{18}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{vmatrix} = \frac{1}{3}$$

$$\bar{x} = \frac{18}{\pi} \iint_R x \sqrt{9x^2+y^2} \, dA = \frac{18}{\pi} \iint_S \frac{u}{3} \sqrt{u^2+v^2} \cdot \frac{1}{3} \, dA = \frac{18}{\pi} \int_0^{\frac{\pi}{2}} \int_0^1 \frac{r\cos\theta}{3} \, r \cdot \frac{1}{3} \cdot r \, dA \quad \cdots = \frac{1}{2\pi}$$

$$\bar{y} \cdots = \frac{3}{2\pi}$$

* Integral factoring (for a, b, c, d constant)

$$\int_a^b \int_c^d f(x)\, g(y)\, dx\, dy = \int_a^b g(y) \left(\int_c^d f(x)\, dx \right) dy = \left(\int_c^d f(x)\, dx \right) \left(\int_a^b g(y)\, dy \right)$$