Lec 34

Stoke's theorem

Recall circulation Greens $\oint_{\partial D} \langle P, Q \rangle \cdot \vec{T}\, ds = \iint_D \text{curl}\, \vec{F} \cdot \hat{k}\, dA$

Notice $\iint_D (\text{curl}\, \vec{F}) \cdot \hat{k}\, dA$ is a surf integral for $\vec{r}(t) = \langle x, y, 0 \rangle$ as S for $(x,y) \in D$

so $\iint_D \text{curl}\, \vec{F} \cdot \hat{k}\, dA = \iint_S \text{curl}\, \vec{F} \cdot d\vec{S}$

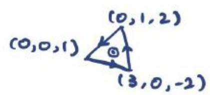
<u>Thm</u> If $\vec{F}$ is a C^1 vec field in $\mathbb{R}^3$ and S is a piecewise smooth oriented surf with boundary ∂S, then

$$\int_{\partial S} \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\, \vec{F} \cdot d\vec{S}$$

Viz. circulation Green's works in $\mathbb{R}^3$

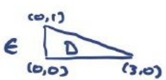
[proof same as Green's]

Ex. $\vec{F} = \langle z, x, y \rangle$ $\quad C$



$\int_C \vec{F} \cdot d\vec{r}$

Parametrise S as $\vec{r}(x,y) = \langle x, y, 1-x-y \rangle$ for $(x,y) \in$



$$\begin{aligned}
\int_C \vec{F} \cdot d\vec{r} &= \iint_S \text{curl}\, \vec{F} \cdot d\vec{S} \\
&= \iint_S \langle 1, 1, 1 \rangle \cdot d\vec{S} \\
&= \iint_D \langle 1, 1, 1 \rangle \cdot (\vec{r}_x \times \vec{r}_y)\, dA \\
&= \iint_D \langle 1, 1, 1 \rangle \cdot (\langle 1, 0, -1 \rangle \times \langle 0, 1, 1 \rangle)\, dA \\
&= \iint_D 1\, dA \\
&= 3/2
\end{aligned}$$

Independence of surface

<u>Thm</u> For vec field with curl curl $\vec{F}$, S and S' oriented surfaces with same boundary $\partial S = \partial S'$,

$$\iint_S \text{curl}\, \vec{F}\, d\vec{S} = \iint_{S'} \text{curl}\, \vec{F}\, d\vec{S}$$ so we only care about curl & boundary

Ex.

S S'

<u>Coro</u> If $\partial S = \emptyset$, then $\iint_S \text{curl}\, \vec{F} \cdot d\vec{S} = 0$

<u>Fact</u> If S closed , $\partial S = \emptyset$

<u>Fact</u> $\text{div}\ \text{curl}\ \vec{F} = 0$