Lec 35

Indep of surf example

Ex. $\vec{F} = \langle z, 2x, 3y \rangle$. Find $\iint_S \text{curl } \vec{F} \, d\vec{S}$ for $\sqrt{9 - x^2 - y^2}$

→ Switch surf

Replace surf with $r=3$ disk. $r(x,y) = \langle x, y, 0 \rangle$ for $D = \{(x,y) \mid x^2+y^2 \leq 9\}$

$\iint_D \text{curl } \vec{F} \cdot (r_u \times r_v) \, dA$

→ Direct compute

$r(x,y) = \langle x, y, \sqrt{9-x^2-y^2} \rangle$ for $D = \{(x,y) \mid x^2+y^2 \leq 9\}$

... more work ...

→ Reverse stoke's to turn it into line integral

$\iint_S \text{curl } \vec{F} \, d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{r}$

$r(t) = \langle 3 \cos t, \ 3 \sin t, 0 \rangle, \quad 0 \leq t \leq 2\pi$

$\int_0^{2\pi} \langle 0, \ 6 \cos t, \ 9 \sin t \rangle \cdot r'(t) \, dt$

Divergence Theorem

Thm Let S be a closed oriented surf in $\mathbb{R}^3$ and E be the region bounded by S,
Let $\vec{F}$ be a C^1 v.f. with domain E. Then:

$$\iint_{S=\partial E} F \cdot \underbrace{d\vec{S}}_{(= \vec{N} \, dS)} = \iiint_E \text{div } \vec{F} \, dV$$

Recall Green's $\int_{\partial S} \vec{F} \cdot \vec{N} \, ds = \iint_S \text{div } \vec{F} \, dA$

Ex. E is $x^2 + 2y^2 + 4z^2 = 1$ $\vec{F} = \langle x, y, z \rangle$

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV$$

$$= \iiint_E 3 \, dV$$

$$= 3 \iiint_E dV$$ ← Use change of var if not using formula

$$= 3 \left(\frac{4}{3} \pi (1) \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{4}}\right) \right)$$

$$= \frac{2\pi}{\sqrt{2}}$$