$\boxed{\text{Lec 36}}$ Vec Calc Review

Set up

$F = \langle P, Q, R \rangle$ or $\langle P, Q \rangle$ in C^1
C piecewise smooth parametrised $\vec{r}(t)$ for $a \le t \le b$
S piecewise smooth surf parametrised $\vec{r}(u,v)$ for $(u,v) \in D$.

Operations

$$\begin{tikzcd} \text{vector field} \arrow[loop left, "\text{curl}"] \arrow[r, bend right, "\text{div}"'] & \text{scalar field} \arrow[l, bend right, "\nabla"'] \end{tikzcd}$$

Conservative check

$$\text{Conservative} \begin{array}{l} \Rightarrow \text{curl } \vec{F} = \vec{0} \\ \Leftrightarrow \text{curl } \vec{F} = \vec{0} \text{ given domain } \vec{F} \text{ simply connected} \\ \Leftrightarrow \vec{F} = \nabla f \text{ for some potential func } f \end{array}$$

Line integral

of scalar field

$$\int_C f\, ds = \int_a^b f(\vec{r}(t))\, \|\vec{r}'(t)\|\, dt$$

of vec field

Circulation $\quad \int_C \vec{F}\cdot\vec{T}\, ds = \int_C \vec{F}\cdot d\vec{r}$

$$\oint_{C=\partial D} \vec{F}\cdot d\vec{r} \xrightarrow{\text{Green's}} \iint_D \text{curl}\, \vec{F}\cdot\hat{k}\, dA$$

$$\int_C F\cdot d\vec{r} \xrightarrow{\text{direct}} \int_a^b \vec{F}(\vec{r}(t))\cdot r'(t)\, dt$$

$$\int_{C=\partial S} \vec{F}\cdot d\vec{r} \xrightarrow{\text{Stoke's}} \iint_S \text{curl}\, \vec{F}\cdot d\vec{S}$$

$$\int_C \nabla f\cdot d\vec{r} \xrightarrow{\text{FTLI}} f(r(b)) - f(\vec{r}(a))$$

Flux $\quad \int_C \vec{F}\cdot\vec{N}\, ds$

$$\int_C \vec{F}\cdot\vec{N}\, ds \xrightarrow{\text{direct}} \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{n}(t)\, dt$$

where $\vec{n}(t) = \langle y'(t), -x'(t)\rangle$

$$\oint_{C=\partial D} \vec{F}\cdot\vec{N}\, ds \xrightarrow{\text{Green's}} \iint_D \text{div}\, \vec{F}\, dA$$

* Surf integrals

of scalar field

$$\int_S f\, dS = \iint_D f(\vec{r}(u,v))\ |\vec{r}_u \times \vec{r}_v|\ dA$$

of vector field, always flux

$$\iint_S \vec{F}\cdot\vec{N}\, dS = \iint_S \vec{F}\cdot dS$$

$$\iint_S \vec{F}\cdot d\vec{S} \xrightarrow{\text{direct}} \iint_D \vec{F}(\vec{r}(u,v))\cdot(\vec{r}_u \times \vec{r}_v)\ dA$$

$$\iint_S \operatorname{curl}\vec{F}\cdot d\vec{S} \xleftrightarrow{\text{Stoke's}} \int_{\partial S} \vec{F}\cdot d\vec{r} \xleftrightarrow[\text{surf}]{\text{indep of}} \iint_{S'} \operatorname{curl}\vec{F}\cdot d\vec{S}$$

where $\partial S = \partial S'$

$$\iint_{S=\partial E} \vec{F}\cdot d\vec{S} \xrightarrow[\text{Thm}]{\text{div}} \iiint_E \operatorname{div}\vec{F}\ dV$$