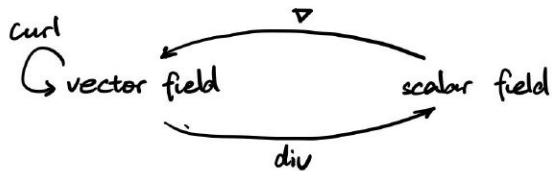


Lec 36 Vec Calc Review

Set up

$F = \langle P, Q, R \rangle$ or $\langle P, Q \rangle$ in C^1
 C piecewise smooth parametrised $\vec{r}(t)$ for $a \leq t \leq b$
 S piecewise smooth surf parametrised $\vec{r}(u, v)$ for $(u, v) \in D$.

Operations



Conservative check

Conservative $\Rightarrow \text{curl } \vec{F} = \vec{0}$
 $\Leftrightarrow \text{curl } \vec{F} = \vec{0}$ given domain \vec{F} simply connected
 $\Leftrightarrow \vec{F} = \nabla f$ for some potential func f

Line integral

of scalar field

$$\int_C f \, ds = \int_a^b f(\vec{r}(t)) \|\vec{r}'(t)\| \, dt$$

of vec field

Circulation $\int_C \vec{F} \cdot \vec{T} \, ds = \int_C \vec{F} \cdot d\vec{r}$

$\oint_{C \rightarrow D} \vec{F} \cdot d\vec{r}$	$\xrightarrow{\text{Green's}}$	$\iint_D \text{curl } \vec{F} \cdot \hat{k} \, dA$
$\int_C \vec{F} \cdot d\vec{r}$	$\xrightarrow{\text{direct}}$	$\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) \, dt$
$\int_{C=\partial S} \vec{F} \cdot d\vec{r}$	$\xrightarrow{\text{Stoke's}}$	$\iint_S \text{curl } \vec{F} \cdot d\vec{S}$
$\int_C \nabla f \cdot d\vec{r}$	$\xrightarrow{\text{FTLI}}$	$f(\vec{r}(b)) - f(\vec{r}(a))$

Flux $\int_C \vec{F} \cdot \vec{N} \, ds$

$\int_C \vec{F} \cdot \vec{N} \, ds$	$\xrightarrow{\text{direct}}$	$\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{n}(t) \, dt$ where $\vec{n}(t) = \langle y'(t), -x'(t) \rangle$
$\oint_{C \rightarrow D} \vec{F} \cdot \vec{N} \, ds$	$\xrightarrow{\text{Green's}}$	$\iint_D \text{div } \vec{F} \, dA$

* Surf integrals

of scalar field

$$\int_S f dS = \iint_D f(\vec{r}(u,v)) |\vec{r}_u \times \vec{r}_v| dA$$

of vector field, always flux

$$\iint_S \vec{F} \cdot \vec{N} dS = \iint_S \vec{F} \cdot d\vec{S} \left\{ \begin{array}{l} \iint_S \vec{F} \cdot d\vec{S} \xrightarrow{\text{direct}} \iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v) dA \\ \iint_S \text{curl } \vec{F} \cdot d\vec{S} \xleftrightarrow{\text{Stoke's}} \int_{\partial S} \vec{F} \cdot d\vec{r} \xleftrightarrow[\text{surf}]{\text{indep of}} \iint_{S'} \text{curl } \vec{F} \cdot d\vec{S} \\ \text{where } \partial S = \partial S' \\ \iint_{S=\partial E} \vec{F} \cdot d\vec{S} \xrightarrow[\text{Thm}]{\text{div}} \iiint_E \text{div } \vec{F} dV \end{array} \right.$$